Beyond Sudoku: Using Puzzles to Develop Students’ Logical-Thinking Skills

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Bridges
Some cells start out with numbers from 1 to 8 inclusive—these are the islands. The goal is to connect all of the islands into a single connected group by drawing a series of bridges between the islands. The bridges must follow certain criteria:

- They must begin and end at distinct islands, traveling a straight line in between (horizontally or vertically)
- They must not cross any other bridges or islands
- At most two bridges connect a pair of islands
- The total number of bridges connected to each island must match the number on that island

Bridges Puzzle 1
These bridge-building challenges come to us courtesy of Puzzles Japan (see the URL above). In each little kingdom below, you are the ruler of the islands. Your goal is to connect them according to the size of their populations—small islands will need only one bridge, while larger ones will need more. All cows must be able to travel to all of the islands.

Here are the rules. The number of bridges should be the same as the number indicated on the island, there can be up to two bridges between two islands. Bridges are horizontal or vertical; they can't cross islands or other bridges, and a continuous path must connect all the islands.
Dominoes Puzzle 1

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### Dominoes Puzzle 3

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**DOMINO PUZZLE**

A standard double-six set of dominoes contains 28 dominoes, with no two alike, as shown in the Domino Chart below. The total number of pips (dots) on each domino ranges from 0 (the double-blank) to 12 (the double-six). In each of these three puzzles, all 28 dominoes have been packed into a 4x7 grid. Puzzle 1 shows a staggered arrangement, but still contains 28 cells. Dominoes are always packed vertically with the lesser number always on top (as illustrated by the 3-5 entered in Puzzle 1).

Your goal is to find each domino’s proper place in the grid, using the numbers outside the grid. The four numbers listed above each column indicate the numbers that will go into the top half of each domino in that column. Likewise, the four numbers listed below each column indicate the numbers that will go into the bottom half of each domino in that column. The rows work the same way—that is, each row’s upper set of seven numbers indicates the numbers that will go into the top half of each domino in that row, the lower set of seven numbers indicates the numbers that will go into the bottom half of each domino in that row.

As you determine the domino locations, you may find it helpful to mark them off the Checklist provided. Realizing that the double-six is the only domino that could have a 0 as a top number, and that the double-blank is the only domino that could have 0 as the bottom number (lower numbers always on top) should help you get started.

**ANSWERS, PAGE 63**

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**Checklist**

0 0 1 2 3 4
1 4 5 6 6
0 1 3 4 5 5
2 3 4 5 6 6
0 0 1 2 3 3
0 1 2 3 6
0 0 1 2 6
2 3 4 5 6

**Puzzle 1**

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Battleships
This puzzle is a solitaire version of the classic paper-and-pencil game of Battleships. The grid represents a section of ocean in which a fleet is hiding. This fleet consists of one battleship (four grid cells in length), two cruisers (three cells each), three destroyers (two cells each), and four submarines (one cell each). The ships may be oriented horizontally or vertically, and no two ships can occupy adjacent grid cells, not even diagonally. The digits along the grid’s perimeter indicate the number of cells in the corresponding rows and columns that are occupied by vessels.

You’ll notice that some “shots” have been taken to start you off. These may show water (indicated by wavy lines), a complete sub (a circle), the bow or stern of a ship (a rounded-off square), or a midsection of a battleship or cruiser (a square).

Battleships Puzzle 1

Battleship
Cruisers
Destroyers
Submarines

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<table>
<thead>
<tr>
<th>Battleships Puzzle 2</th>
<th>Battleships Puzzle 3</th>
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<tbody>
<tr>
<td><img src="image1" alt="Battleships Puzzle 2" /></td>
<td><img src="image2" alt="Battleships Puzzle 3" /></td>
</tr>
</tbody>
</table>

- **Battleship**: 
  - ![Battleship](image1)

- **Cruisers**: 
  - ![Cruisers](image1)

- **Destroyers**: 
  - ![Destroyers](image1)

- **Submarines**: 
  - ![Submarines](image1)
Battleships
The six puzzles on this page are solitaire versions of the classic paper-and-pencil game of Battleships. Each grid represents a section of ocean in which a fleet is hiding. This fleet consists of one battleship (four grid cells in length), two cruisers (three cells each), three destroyers (two cells each), and four submarines (one cell each). The ships may be oriented horizontally or vertically, and no two ships can occupy adjacent grid cells, not even diagonally. The digits along the grid’s perimeter indicate the number of cells in the corresponding rows and columns that are occupied by vessels.

You’ll notice that some “shots” have been taken to start you off. These may show water (indicated by wavy lines), a complete sub (a circle), the bow or stern of a ship (a rounded-off square), or a midsection of a battleship or cruiser (a square).
The six puzzles on this page are variations of the classic paper-and-pencil game of Battleships. Each grid represents a section of ocean in which a fleet is hiding. The fleet consists of one battleship (four cells long), two cruisers (three cells each), three destroyers (two cells each), and four submarines (one cell each). The ships may be oriented either horizontally or vertically, and no two ships can occupy adjacent grid cells, nor even diagonally. The digits along the grid's perimeter indicate the number of cells in the corresponding row and columns that are occupied by vessels.

You'll notice that some "ships" have been taken to start you off. These may show water indicated by waves, a complete ship (shaded), the bow or stern of a ship (a rounded-off square), or a middle of a battleship or cruiser (square). The puzzles get harder as you go. Can you reach the rank of admiral by locating all six fleets?

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Fences
Each of the puzzles below represents a plot of land; your job is to section off part of the plot with a
continuous fence. To build a fence, draw a line connecting one fence post to the next (horizontally or
vertically—but never diagonally), continuing until you return to where you started, forming a single
closed loop. The digits in the puzzle indicate the number of fence segments you must draw along the
sides of the numbered square.

Fences Puzzle 1

Fences Puzzle 2
## Fences

Each of the puzzles below represents a plot of land; your job is to section off part of the plot with a continuous fence. To build a fence, draw a line connecting one fence post to the next (horizontally or vertically—but never diagonally), continuing until you return to where you started, forming a single closed loop. The digits in the puzzle indicate the number of fence segments you must draw along the sides of the numbered square.

<table>
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</table>
These puzzles are also known as Fences, and they come to us courtesy of Puzzle Japan (see the URL above right). Your goal is to fence off a part of each puzzle with a closed circuit. Just draw a line from dot to dot (horizontally or vertically—never diagonally), continuing until you return to your starting dot. Each digit indicates the number of fence segments you must draw along the sides of that numbered square. The example shows a solved puzzle.
Resources

Bridges
- **HASHI: The Bridges Puzzle** by Alastair Chisholm (2006)
- Occasional **GAMES** and **GAMES World of Puzzles** magazines
- A freeware game “Hashiwokakero” available for PC computers at the Mindgames website: http://ihsan.biz/math.html
- Online applets:
  - Millions of computer-generated Bridges puzzles at http://www.puzzle-bridges.com/
  - A few hand-created Hashi-o-Kakero puzzles (more visually interesting grids with rotational symmetry of islands) at http://www.redbrick.dcu.ie/~kevmcg/bridges/

Dominoes
- Occasional **GAMES** and **GAMES World of Puzzles** magazines
- A shareware game “Domino Delimma” available for PC computers at the Sapphire Games website: http://www.sapphregames.com/dominoes?
- A free widget for Mac computers named “Dominogy” by Gandreas Software. A full version with 300 puzzles is available to buy: http://projects.gandreas.com/dominogy/index.html
- Links to other dominoes puzzles and games at: http://www.dominogames.com/domino-puzzles.html

Battleships
- **Every** issue of **GAMES** and **GAMES World of Puzzles** magazines
- The Mountain Vista Software website contains:
  - **Fathom It!** - A downloadable PC version of Battleships in both a demo version (free with a number of sample games of different sizes) and a full version with hundreds of puzzles at: http://www.mountainvistasoft.com/index.htm
  - The “Battleship Puzzle Omnibus” - a detailed compendium of Battleships puzzle variants and links to Battleships puzzles used in various competitions (e.g., the World Puzzle Championship), monographs, and analyses of Battleships puzzles.
  - **Yubotu: Sink the Fleet in These Addictive Battleship Puzzles** (2006) and **Sit & Solve Travel Battleship Puzzles** (2006) by Peter Gordon and Mike Shenk

Fences
- Occasional issues of **GAMES** and **GAMES World of Puzzles** magazines
- A very challenging online version called “Kwon-Tom Loop” where a new puzzle is given each day. Solvers are timed and a leader board presents fastest score. “Easiest” puzzles appear on Mondays and the puzzles get more difficult and larger through the week. At: http://www.kwontomloop.com/index.php
- Millions of computer-generated Slither Link puzzles at http://www.puzzle-loop.com/

Other online sources for language-independent puzzles
- World Puzzle Federation – Team USA at: http://wpc.puzzles.com/
- Math Puzzle site – lots of advanced mathematics and news about math puzzles at: http://www.mathpuzzle.com/

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